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Complex pulsed field magnetization behavior and Walker breakdown in a NiFe thin-film

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The magnetization behavior of a Permalloy thin-film (nominally $\text{Ni}_{81}\text{Fe}_{19}$) was investigated as a function of combined quasistatic and pulsed magnetic fields measured using magneto-optic Kerr effect magnetometry. We observed complex field dependent switching behavior that depends on the relative contributions to the total field of the quasistatic and pulsed fields. As the pulsed field amplitude was increased, complex switching behavior occurs for total fields in excess of the coercive field. A simple phenomenological domain wall propagation model suggests a qualitative understanding of this complex behavior based on Walker breakdown of the domain wall motion occurring in the Permalloy thin-film. © 2010 American Institute of Physics.

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I. INTRODUCTION

The magnetization behavior of magnetic thin-films is of basic interest and also relevant to technological applications such as sensors, electronic components, and recording media. Magnetization reversal can take place through various processes from domain wall propagation¹ to quasicohere rotation.² The dominant processes are determined by the energetics of the system that includes a number of contributions such as anisotropy and exchange, they are also dependent upon the timescale and amplitude of the applied magnetic field and can be thermally assisted.³

For Permalloy thin-films (compositions around $\text{Ni}_{80}\text{Fe}_{20}$) the magnetization process typically involves nucleation and propagation of domain walls throughout the film.⁴ The domain wall propagation velocity in Permalloy thin-films has been determined experimentally by several groups^{4–7} and it was shown that the velocity increases monotonically with field and the field dependence of the wall velocity may be divided into two regimes: a high-field regime controlled by gyromagnetic damping; and a low-field regime where thermally activated depinning contributes to the propagation time. In contrast, theoretical studies^{8,9} and simulations^{10,11} have shown that in the absence of pinning the domain wall velocity can increase approximately linearly with field up to a critical field beyond which the domain wall structure is no longer stable; in this case both the wall structure and wall motion become complex including periods of retrograde motion which has been termed Walker breakdown. Experiments on out-of-plane bubble material have observed Walker breakdown in the domain wall dynamics of thin-films.¹² More recently, experimental velocity measurements on a 200 nm wide and 5 nm thick Permalloy nanowire^{11,13} hinted at the presence of Walker breakdown in the field dependence of the domain wall velocity and more recent measurements on a 600 nm wide and 20 nm thick Permalloy nanostructure¹⁴ showed a peak in the field dependence of the domain wall

velocity indicative of Walker breakdown; similar behavior has subsequently been observed by others in Permalloy nanowires.¹⁵ Therefore, in principle the magnetization behavior of thin-films, such as NiFe, in pulsed magnetic fields may result in complex field dependence of the magnetization arising from the complexity of the domain wall dynamics.

Here the magnetization switching behavior in an unstructured Permalloy thin-film was studied with a combination of pulsed and quasistatic magnetic fields applied along the same axis. The results are interpreted using a phenomenological model based upon the depinning and propagation of a single domain wall, where the field dependence of the domain wall propagation includes Walker breakdown.

II. EXPERIMENTAL DETAILS

The sample investigated was fabricated by thermal evaporation through a shadow mask onto a on a 5×5 mm silicon substrate with a 500 nm thick hydrothermally oxidized SiO_2 surface layer. The evaporation base pressure was approximately 10^{-7} Torr and the deposition rate of the order of 1 Å s^{-1} . The sample consisted of a 400 μm wide microstrip line made of a 30 nm thick aluminum layer with a 15 nm thick $\text{Ni}_{81}\text{Fe}_{19}$ layer deposited directly on top through the same shadow mask without breaking the vacuum and in the absence of a magnetic field. The layer thicknesses were determined during deposition using an *in situ* quartz crystal oscillator that was externally calibrated to thicknesses determined from x-ray reflectivity measurements.

The silicon substrate base was fixed to an earthed sample holder and the microstrip on top of the substrate was connected at each end to the center pins of 50 Ω coaxial connectors using conductive silver paint. The sample holder was positioned within the poles of an electromagnet supplying a quasistatic field. The microstrip was connected via the coaxial connectors to an impedance matched pulsed current generator circuit. Flat-topped current pulses were used to produce pulsed magnetic fields of widths from 10 ns up to 1 μs across the microstrip line. Figure 1 shows a schematic

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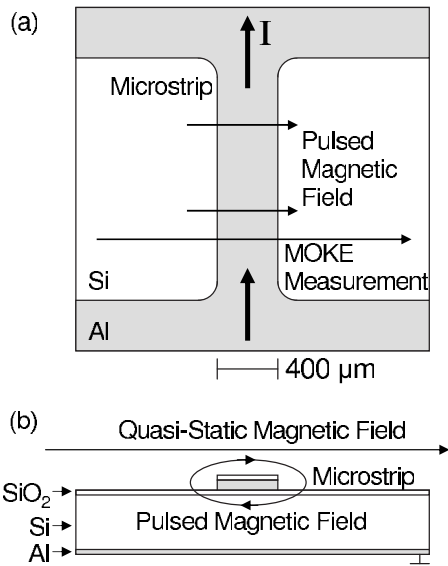


FIG. 1. (a) Plan view schematic illustration of the sample also showing the externally applied quasistatic field and the induced pulsed field from the microstrip line and (b) end view schematic illustration showing the cross section of the sample, the Permalloy thin-film is the top layer on the microstrip.

of the sample showing the microstrip structure and the orientation of the applied magnetic fields. In this study the effect on magnetization reversal of combining both pulsed and quasistatic magnetic fields was investigated.

A focused longitudinal magneto-optical Kerr effect (MOKE) optical system¹⁶ was used to determine the magnetization state resulting after the application of the total field along the axis of the applied fields. The laser spot focused onto the sample was elliptical and around 10 μm in size.

The field-current relationship for the microstrip was calibrated by passing a small dc current through the microstrip. This introduced a small static field shift to the electromagnet driven MOKE loop for the sample that was used to calibrate the field-current relationship in the microstrip.

The quasistatic field applied from the electromagnet consisted of a sinusoidally varying ac field at 27 Hz with an amplitude of 13 Oe, sufficient to saturate the magnetization of the sample. This was combined with a dc offset allowing an effective bias field to be added to the pulsed fields which is equal to $H_{bias} = \max(H_{ac}) + H_{dc}$. This 27 Hz sinusoid field was quasistatic with respect to the 10 ns to 1 μs duration of the pulsed magnetic fields. Each measurement involved averaging the MOKE signal over several hundred field cycles; therefore, the magnetization changes obtained are averages of any stochastic switching behavior. The effective magnetization was derived by normalizing the Kerr signal data after removing a linear field dependent background contribution attributed to secondary magneto-optical effects that continued at fields higher than the saturation field of the sample.

For this study the dc offset was adjusted until the sum of the offset and the maximum of the 27 Hz field was not large enough to cause switching to positive magnetization, but was sufficient to reset the sample to negative saturation magnetization after each field cycle. Pulses of width from 10 ns to 1 μs with rise times of <1 ns were triggered at the quasi-

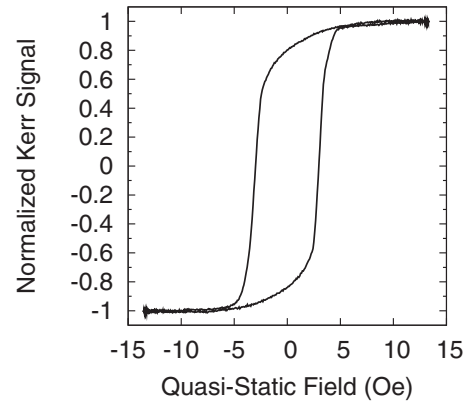


FIG. 2. Normalized MOKE hysteresis loop for the Permalloy thin-film measured with a magnetic field at 27 Hz.

static field maximum using a digital to analog converter which synchronizes the pulsed field triggering and the quasistatic field. The jitter on the pulsed field circuit switching is negligible on the timescale of the quasistatic field.

The magnetization behavior for the thin-film was obtained by recording the hysteresis loops for a range of pulsed field magnitudes at fixed bias fields. The change in magnetization is small if switching was mostly unsuccessful and large if the sample was switched during every field cycle. The relative contributions to the total field of the pulsed and bias components were varied and the experiment was repeated. Both positive and negative bias fields were investigated and were repeated for a range of different pulse widths.

To aid interpretation of the results obtained, a simple model was developed where magnetization switching takes place by field driven domain wall propagation. The model takes into account the domain wall propagation characteristics resulting from both the short duration pulsed fields and the bias fields used in the experiment.

III. RESULTS AND DISCUSSION

Figure 2 shows the quasistatic (27 Hz) magnetization behavior of the Permalloy thin-film sample measured using the MOKE system. The sample shows ferromagnetic behavior with a quasistatic coercivity of 3 Oe, which is typical for films of this material and thickness,¹⁷ and the material reaches saturation around 10 Oe. The film has a high remanence ratio of over 0.8 indicating a large magnetization component along the applied field axis and MOKE measurement axis. Note that the sample fabrication with microstrip line geometry in this investigation restricts pulsed fields to be applied only along one axis.

Before application of the pulsed field, the quasistatic 27 Hz ac signal and dc offset from the electromagnet were used to initialize the magnetization state before the pulsed magnetic field was triggered. As described earlier, the bias field was adjusted until rapid switching of the film through the coercive point was no longer achieved, as shown in Fig. 3(a). By applying a 1 μs pulsed magnetic field coinciding with the maximum positive quasistatic field, partial or complete magnetization switching was restored, depending upon the amplitude of the pulsed field, as shown in Figs. 3(b)–3(d).

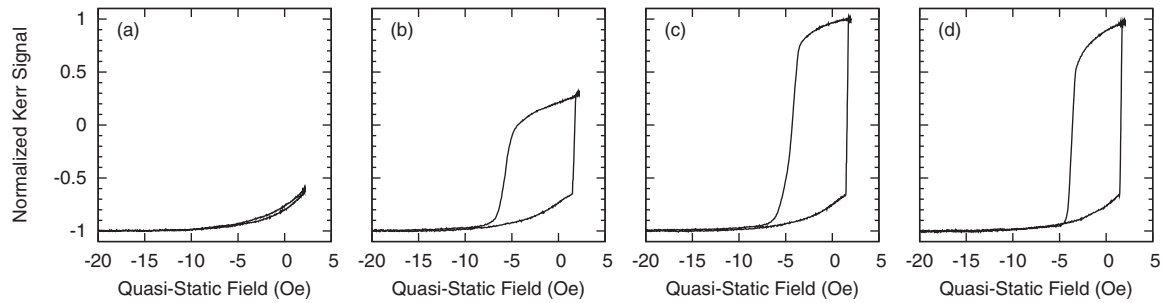


FIG. 3. MOKE loops for the Permalloy thin-film sample where the 27 Hz drive field has been combined with a dc offset field to provide a bias field. (a) The bias field alone is insufficient to cause switching of the magnetization to positive fields. While the addition of a pulsed field of (b) 2.8 Oe, (c) 3.7 Oe, and (d) 12.2 Oe, respectively, triggered at the quasistatic positive field maximum leads to magnetization switching.

The normalized change in magnetization from negative saturation to the maximum magnetization in Fig. 3 is shown as a function of pulsed field amplitude in Fig. 4(a). The effect of different bias fields on the switching behavior is also shown in this figure. The same data is replotted in Fig. 4(b) as a function of the total field (the sum of both the pulsed and bias fields). The data sets all initially rise at the same total field (within a spread of <0.5 Oe, which is within the resolution of the calibration for the electromagnet) showing that for a fixed pulse width the switching field for the thin-film is dependent on the total field combining the relative contributions from the pulsed and bias fields.

The changes in pulsed field magnetization switching as a function of the bias field are complex. At the largest positive

bias field simple steplike field dependent switching behavior occurs. However, with a decreased bias, the rapid rise in magnetization with field is followed by a fall in magnetization level at higher fields. When the bias is negative the switching becomes more complex as the magnetization rises to a peak then falls significantly and rises again as the pulsed field level increases further. For the small positive bias and the negative bias fields the final magnetization state is one of incomplete switching even though the maximum in the total field during the pulse is in excess of the switching field.

Figure 5 shows the change in magnetization for the sample as a function of total applied field for pulse widths of about 1 μ s, 500 ns, and 10 ns and for both positive and

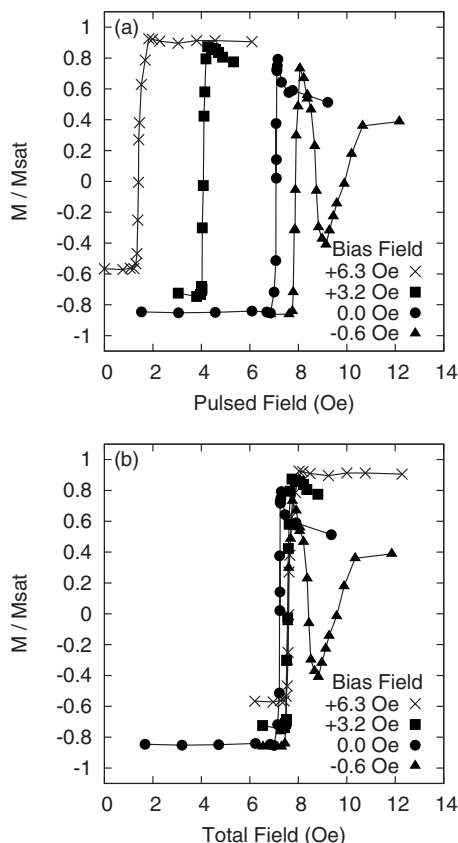


FIG. 4. The change in normalized magnetization switching as a function of (a) the pulsed field amplitude and (b) the total field amplitude for 1 μ s long field pulses.

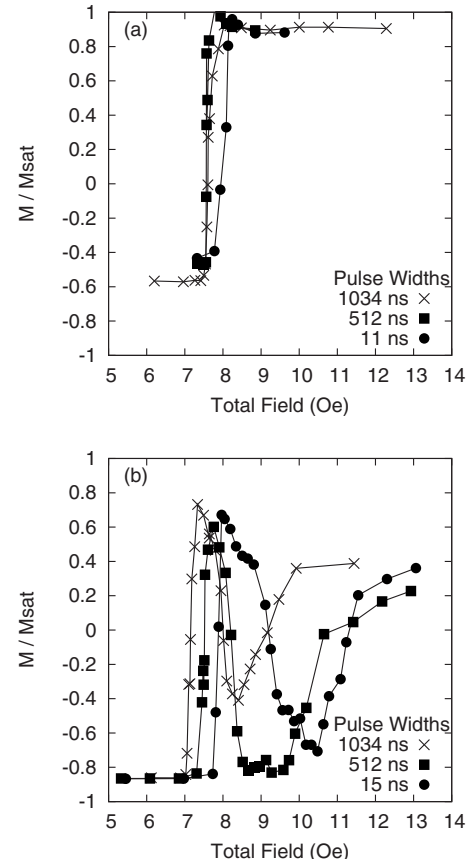


FIG. 5. The dependence on pulse width of the change in normalized magnetization as a function of the total field for (a) positive bias fields and (b) negative bias fields.

negative bias fields. For positive bias fields, as shown in Fig. 5(a), the pulse width has little effect on the switching behavior taking place, with perhaps a small increase in the switching field at the shortest pulse timescale. In contrast, with negative bias fields, Fig. 5(b), there is a significant pulse width dependence of the magnetization switching behavior and it can be seen that the field at which the initial magnetization switching occurs decreases with increasing pulse width, which is consistent with thermally activated pulsed field switching.¹⁸ The pulse width has a negligible effect on the magnetization for total fields below the switching field and on the peak in magnetization at the switching field.

Figures 4 and 5 show the change in magnetization resulting from the combination of the pulsed and bias fields. The bias field is responsible for a small reversible change in magnetization from negative saturation [Fig. 3(a)] which is constant with bias field. This is represented by the horizontal data at low fields in Figs. 4 and 5 for total fields below the switching field.

A simple model has been developed to aid interpretation of the magnetization switching behavior observed in the thin-film. The model considers a region within a magnetic thin-film with dimensions of the same order as the MOKE laser spot. The magnetization within the model is represented by the areas of two opposite magnetized domains separated by a single domain wall. In the model, switching takes place as this domain wall depins and propagates across the modeled region increasing the area of one domain with respect to the other. The reversible components of the magnetization observed in the experimental results are small and constant at each bias field and are, therefore, neglected in this model, although it is recognized that this is a simplification.

Figure 6(a) illustrates schematically the modeled region within a section of magnetic thin-film. Domain wall density in NiFe thin-films has been observed elsewhere to be dependent upon film thickness and have spacing of the order of a few micrometers,¹⁹ so modeling magnetization switching by the propagation of a single domain wall is a reasonable assumption for modeling on the 10 μm scale. Furthermore, the magnetization behavior of the macroscopic thin-film can be represented by the collective behavior of many such single domain walls across the thin-film. This single wall model is simplified, but based on reasonable assumptions and at most only a few domain walls are likely to contribute to magnetization switching in the area probed by the laser spot of the focused MOKE.

Magnetic field driven domain wall behavior has been modeled following Ferré²⁰ and Atkinson *et al.*¹¹ who use a linear field dependent domain wall velocity:

$$v = (\gamma\Delta/\alpha)H, \quad (1)$$

dependent on the gyromagnetic ratio γ , the domain wall width Δ , and damping coefficient α . Propagation at this velocity occurs for the remaining time of the field pulse following the wait time needed for the thermally activated depinning of the wall. This depinning time, τ , is obtained from a simple Arrhenius process;²¹

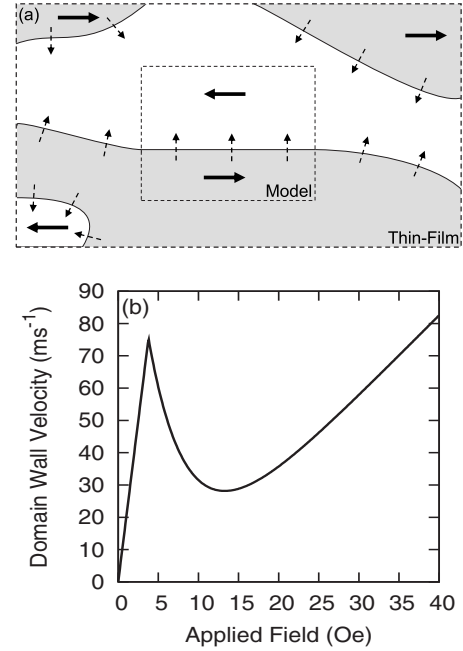


FIG. 6. (a) Illustration of a section of magnetic thin-film showing a modeling area comparable in size to the MOKE laser spot. Micrometer sized domains with magnetization indicated with solid arrows are separated by domain walls. During a field driven switching event, these domain walls propagate in the direction indicated by the dashed arrows. (b) The parameterized field velocity relationship derived for the experimental results obtained by Beach *et al.* (Ref. 14).

$$\tau = \tau_0 e^{\Delta E/k_B T}, \quad (2)$$

where $\tau_0 = 1 \times 10^{-9}$ s represents the inverse attempt frequency²² and $k_B T$ the thermal energy at $T = 300$ K. The energy barrier for the depinning processes is modeled as;

$$\Delta E = E_0(1 - H/H_0)^\beta, \quad (3)$$

from the activation of a single process,²³ using a zero temperature energy barrier, $E_0 = 3 \times 10^{-20}$ J (comparable with experimental values obtained by Lendcke *et al.*²⁴), and zero temperature switching field $H_0 = 30$ Oe, which have been iterated so that the modeled behavior compares with experimental results obtained. The exponent β is set to a reasonable value of 1.5.²⁵

These energetic relationships are originally derived from switching in a single particle by coherent rotation. Modeling domain wall propagation as a single energy barrier process is a simplification, but this is reasonable because the details of the profile of the energy barrier are negligible in comparison to its height which is much greater than $k_B T$.

At low fields, the energy barrier associated with the domain wall depinning for this reversal process is not overcome resulting in unsuccessful switching. When the total field is increased until it exceeds the switching field, the energy barrier can be overcome resulting in switching of the sample.

The model returns the normalized change in magnetization, M , from negative saturation after the application of the pulsed and bias field for a propagation time followed by the bias field alone for a set measurement time;

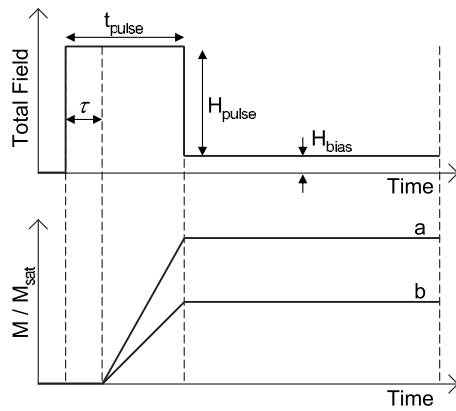


FIG. 7. Schematic illustration of the applied field sequence for the model. The pulsed field in combination with the bias field is applied. The wall moves after the depinning time for the pulse duration minus the depinning time. This is followed by a measurement time in which only the bias field is applied. (a) Magnetization is switched for large domain wall velocities and (b) this switching is limited by domain wall velocity resulting in a partially switched state.

$$M = \frac{[v(H_{\text{pulse}} + H_{\text{bias}})^*(t_{\text{pulse}} - \tau) + v(H_{\text{bias}})^*t_{\text{meas}}]*2}{w - 1}. \quad (4)$$

Here $w = 30 \mu\text{m}$ is the model width (of the same order as the MOKE laser spot), $v(H)$ the domain wall velocity, dependant on the fields H_{pulse} and H_{bias} the pulsed and bias fields, respectively. The pulse duration t_{pulse} and measurement time $t_{\text{meas}} = 10 \mu\text{s}$ (comparable with the measurement field cycle) and τ the depinning time from Eq. (2). Figure 7 provides a schematic timing diagram of the applied fields and the resulting magnetization change they induce.

Here the model has been developed to include the effect of Walker breakdown by replacing the linear field dependent velocity from Eq. (1) with a parameterized field dependent velocity relation [see Fig. 6(b)] obtained from the experimental field dependent velocity data obtained by Beach *et al.*¹⁴ This data shows similar behavior to that obtained from micromagnetic simulations on planar nanowires that show a peak in the field dependent velocity due to Walker breakdown.^{11,26} With low applied fields the domain wall velocity increases approximately linearly with field, and when the field reaches the Walker breakdown field, complex dynamical behavior occurs within the domain wall resulting in a lower averaged domain wall velocity. As the field is increased further, there is a recovery in the wall velocity beyond the Walker breakdown regime.

By introducing the effect of Walker breakdown on the domain wall motion, this model can be used to qualitatively explain the magnetization switching behavior observed for different bias fields. Figure 8 shows modeled results for both positive and negative bias fields. The model shows some qualitative agreement with the experimental data. For positive bias fields the model shows simple field dependent switching where the magnetization increases rapidly at the switching field to a final value. In contrast, the magnetization switching behavior for negative bias fields is complex. The magnetization rises initially to a peak before falling to a minimum as the field increases further and then increases again at higher fields. As expected, the switching field sepa-

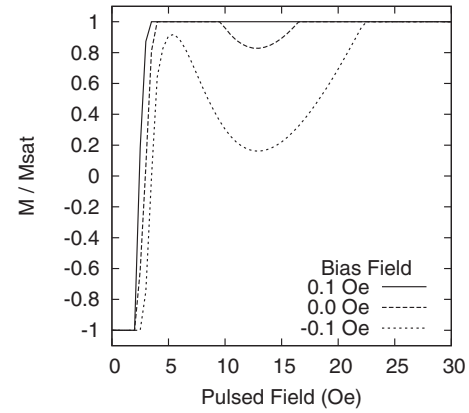


FIG. 8. Modeled behavior showing the effect of the bias field upon the pulsed field magnetization switching behavior of Permalloy thin-film with a $1 \mu\text{s}$ pulse. The complex peaked switching behavior arises from the inclusion in the model of Walker breakdown in the field dependent velocity behavior.

rates the unsuccessful switching at low fields from successful switching at higher fields. However, for negative bias fields a further increase in pulsed field magnitude shifts the domain wall motion from its initial linear regime to the Walker breakdown regime. Here, the larger drive fields result in lower time averaged domain wall velocity and longer transit times. Hence the timescale for switching is increased and saturation may not be achieved before the pulsed field terminates. If the magnetization switching is not completed during the application of the pulse field the domain wall is still present and the subsequent magnetization behavior depends upon the bias field. If the bias field is positive the depinned domain wall continues to move and saturation can be achieved. For negative bias fields the domain wall can travel in the reverse direction and results in a less switched state.

Another feature in common between experimental and modeled results is the total field for switching. This field is independent of the relative contribution from pulsed and bias fields, in agreement with experimental results. In the model we ignore reversible magnetization rotation taking place within the samples. This accounts for the difference in the change in magnetization between experimental and modeled results for fields below the switching field. In the model, no change occurs to the magnetization as there is not enough energy to overcome the depinning energy and no reversible contributions are included in the model.

The effect of the width of the field pulse was also investigated using the model (see Fig. 9) for a constant bias field of -0.1 Oe . Modeled results show some features in common with the complex behavior found in experimental results. The final magnetization state for total fields below the switching field is independent of the pulse width and this switching field reduces with increasing pulse width. This is consistent with thermally activated switching processes.

IV. CONCLUSIONS

The magnetization behavior of a Permalloy thin-film has been investigated with a combination of quasistatic and pulsed magnetic fields. With a larger positive bias field, simple switching behavior is found where magnetization

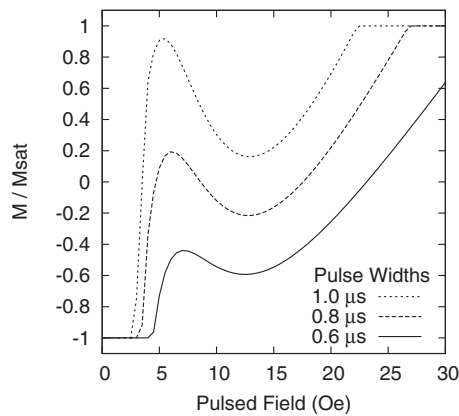


FIG. 9. Modeled behavior showing the effect of the pulse width upon the pulsed field magnetization switching behavior of a Permalloy thin-film with a -0.1 Oe bias field. The complex switching behavior is affected by pulse width and the switching field is consistent with thermally activated pulsed field switching.

switching takes place over a small field range and saturation is achieved with increasing of the field. In contrast, when the bias field is negative the magnetization behavior is more complex showing first a rapid rise in magnetization around the switching field. However, at higher fields switching measurements show reduced magnetization switching while for yet higher fields the level of magnetization switching increases.

This behavior has been modeled using a domain wall propagating in a region of magnetic thin-film on the $10\ \mu\text{m}$ scale with a field dependent propagation that includes Walker breakdown. The model shows that for modest fields around the coercivity the magnetization switches fully as the domain walls can successfully propagate across the full width of the model in the duration of the pulse. For larger fields, Walker breakdown reduces the time averaged domain wall velocity so that the wall no longer reaches the far side of the model before the pulse is terminated. At this point the bias field then drives the domain wall motion. For larger positive bias fields the wall is driven to complete the magnetization switching while a negative bias field reverses the wall mo-

tion resulting in a less switched state. This simple model suggests a qualitative understanding of this complex behavior indicating that Walker breakdown of the domain wall propagation is limiting the magnetization switching behavior observed in Permalloy thin-film.

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